THE COLLEGE FINALS

The Finals will be conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has aQempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

The last person remaining wins an additional $75 and will be crowned the **Integration Champion**!

INTEGRAL #1

READY,
GET SET,...

2:30

2:30

INTEGRAL #1

$$\int \frac{\ln x}{x^2}\,dx$$

[integrate by parts: $u = \ln x$, $du = \frac{1}{x}\,dx$; $dv = \frac{1}{x^2}\,dx$, $v = -\frac{1}{x}$]

$$= -\frac{\ln x}{x} + \int \frac{1}{x^2}\,dx = -\frac{\ln x}{x} - \frac{1}{x} + C$$

$$= \boxed{-\frac{1 + \ln x}{x} + C}$$

INTEGRAL #2

READY,
GET SET,...

2:30

INTEGRAL #2

$$\int \left(e^{x} + e^{-x}\right)^{2} dx$$

2:30

INTEGRAL #2

$$\int \left(e^{x} + e^{-x} x x\right.$$

INTEGRAL #3

READY,
GET SET,...

2:30

INTEGRAL #3

$$\int e^{2x} \sqrt{1 + e^{2x}}\, dx$$

2:30

INTEGRAL #3

$$\int e^{2x} \sqrt{1 + e^{2x}}\, dx$$

$$= \frac{1}{2} \int \sqrt{u}\, du \quad \left[u = 1 + e^{2x}; \quad du = 2e^{2x}\, dx\right]$$

$$= \frac{u^{3=2}}{3} + C$$

$$= \boxed{\frac{\left(1 + e^{2x}\right)^{3=2}}{3} + C}$$

INTEGRAL #4

READY,
GET SET,...

2:30

2:30

INTEGRAL #4

$$\int \frac{\ln \sqrt{x}}{x}\,dx$$

$$= \int \frac{\ln x^{1=2}}{x}\,dx = \frac{1}{2}\int \frac{\ln x}{x}\,dx$$

$$= \frac{1}{2}\int u\,du \qquad \left[u = \ln x; \quad du = \frac{1}{x}\,dx\right]$$

$$= \frac{u^2}{4} = \boxed{\frac{(\ln x)^2}{4} + C}$$

INTEGRAL #5

READY,
GET SET,...

2:30

INTEGRAL #5

$$\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}\,dx$$

2:30

INTEGRAL #5

$$\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}\,dx$$

$$= \frac{1}{2}\int \frac{1}{u}\,du \qquad \left[u = e^{2x} + e^{-2x};\quad du = \left(2e^{2x} - 2e^{-2x}\right)dx\right]$$

$$= \frac{1}{2}\ln u + C$$

$$= \boxed{\frac{\ln\left(e^{2x} + e^{-2x}\right)}{2} + C}$$

INTEGRAL #6

READY,
GET SET,...

2:30

INTEGRAL #6

$$\int x \sec^2 x \, dx$$

2:30

INTEGRAL #6

$$\int x \sec^2 x \, dx$$

$$\left[\text{integration by parts: } \begin{array}{rcl} u &=& x \\ du &=& dx \end{array} ; \quad \begin{array}{rcl} dv &=& \sec^2 x \, dx \\ v &=& \tan x \end{array} \right]$$

$$= x \tan x - \int \tan x \, dx$$

$$= \boxed{x \tan x + \ln|\cos x| + C}$$

INTEGRAL #7

READY,
GET SET,...

2:30

INTEGRAL #7

$$\int x\left(1+\frac{1}{x}\right)^{3}\,dx$$

2:30

INTE

$$\int x\left(1+\frac{1}{x}\right)^3 dx$$

$$= \int x\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right) dx$$

$$= \int \left(x+3+\frac{3}{x}+\frac{1}{x^2}\right) dx$$

$$= \boxed{\frac{x^2}{2}+3x+3\ln x-\frac{1}{x}+C}$$

INTEGRAL #8

READY,
GET SET,...

2:30

INTEGRAL #8

$$\int \cos^5 x \, dx$$

2:30

INTEGRAL #8

$$\int \cos^5 x \, dx$$

$$= \int (\cos^2 x)^2 \cos x \, dx = \int (1 - \sin^2 x)^2 \cos x \, dx$$

$$= \int (1 - u^2)^2 \, du \quad [u = \sin x; \quad du = \cos x \, dx]$$

$$= \int (u^4 - 2u^2 + 1) \, du = \boxed{\frac{\sin^5 x}{5} - \frac{2 \sin^3 x}{3} + \sin x + C}$$

INTEGRAL #9

READY,
GET SET,...

2:30

INTEGRAL #9

$$\int \frac{x^2 + 1}{(x + 1)^2}\, dx$$

2:30

INTEGRAL #9

$$\int \frac{x^2 + 1}{(x + 1)^2} \, dx$$

2:30

INTEGRAL #11

READY,
GET SET,...

2:30

INTEGRAL #11

$$\int \frac{1}{\sqrt{x^2 - 1}}\, dx$$

2:30

$$\int \frac{1}{\sqrt{x^2 - 1}}\, dx$$

$$\left[x = \sec \quad ; \quad dx = \sec \quad \tan \quad d \; ; \quad \sqrt{x^2 - 1} = \tan \right]$$

$$= \int \frac{\sec \quad \tan}{\tan}\, d \quad = \int \sec \quad d$$

$$= \ln\left|\sec \quad + \tan \right| + C = \boxed{\ln\left|x + \sqrt{x^2 - 1}\right| + C}$$

INTEGRAL #12

READY,
GET SET,...

2:30

INTEGRAL #12

$$\int \sin^2 x \cos^2 x \, dx$$

2:30

[redacted]

$$\int \sin$$